Probability density function and ''plus'' and ''minus'' structure functions in a turbulent channel flow

Miguel Onorato

Dipartimento di Fisica Generale, Università di Torino, Via Pietro Giuria 1, 10125 Torino, Italy

Gaetano Iuso

Dipartimento di Ingegneria Aerospaziale, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

We consider the statistical properties of the longitudinal velocity increments in a turbulent channel flow at different distances from the wall. The probability density function (PDF) of the velocity difference of the streamwise component near the wall are found to be, especially at small scales, strongly skewed, showing a very long left tail. We consider ''plus'' and ''minus'' structure functions and compute separately the statistics for the right and left part of the PDF. It is found that the relative scaling exponents for the right tail are less affected by the presence of the wall and their values are consistent with the ones found in experiments in homogeneous and isotropic turbulence. A simple phenomenological model that explains the results obtained is also given.

Recently, a number of papers on the intermittency properties of the streamwise velocity component in turbulent channel flow have been published [1–5]. A major finding, consistent with all the work published, is that a dependence of the scaling exponent, ζ_p, on the distance from the wall is observed through the extended self-similarity (ESS) [6]. A similar result has been obtained from experimental data in a magnetically confined turbulent fusion plasma [7]. Moreover in [1], using an eduction method based on the wavelet transform, it was found that strong velocity gradients are responsible of the increase of intermittency close to the wall. In none of the cited papers the asymmetry of the probability density functions (PDF's) of the longitudinal velocity increments has been investigated. A different approach that takes into account the contribution of anisotropy has been introduced by Arad *et al.* [8]. They decompose the structure functions into their irreducible representation of the SO(3) symmetry group and their finding is that the isotropic contribution has a wide scaling region and is universal. Their analysis has been performed at the center and at one fourth of the channel height. Nevertheless, it has to be pointed out that close to the wall, besides effects of anisotropy, a different dynamics should occur due to the presence of the wall. One of its major effects is the creation of coherent structures such as ''low'' and ''high'' speed streaks and streamwise vortices. Their dynamics, as anticipated in [1], must, in some way, influence the shape of the PDFs and the values of the relative scaling exponents.

In this Rapid Communication we focus our attention on the asymmetry of the PDFs of the longitudinal velocity increments. The analysis is carried out by means of the ''plus'' and ''minus'' structure functions at different distances from the wall. We provide tentative evidence that the statistics of the positive velocity increments is less affected by the presence of the wall. Moreover, we give a possible explanation of this behavior in terms of the dynamic of coherent structures concentrated in the near wall region. The ESS is used as a statistical method for characterizing and comparing data at different distances from the wall.

The experiment has been carried out in a rectangular cross section channel, 7 m long, 70 mm high, and 300 mm wide, arranged in five plexiglas modules. The Reynolds number based on the mean velocity at the center of the channel and on the channel half height was 10 800 which corresponds to $Re_\tau = 510$ (Re_τ is the Reynolds number based on the friction velocity). Validation of the data set for basic statistical quantities such as mean, rms, skewness and flatness are shown in [1]. Data have been taken at different positions in the y^+ coordinate that is the coordinate normal to the wall (superscript $+$ indicates that quantities are in wall units). We concentrate our analysis in the buffer layer ($y^+ = 7$, 10, 15, 28, and 35) and almost at the center of the channel ($y^+ = 218$ and 310). In Fig. 1 we show the PDFs of the streamwise velocity difference at $y^+ = 218$ for different values of the scale r^+ (variation in time are converted into space variation with the Taylor hypothesis). The PDF continually changes its character as the separation r^+ varies: from a quasi-Gaussian curve, when r^+ is comparable to large scales, to some expo-
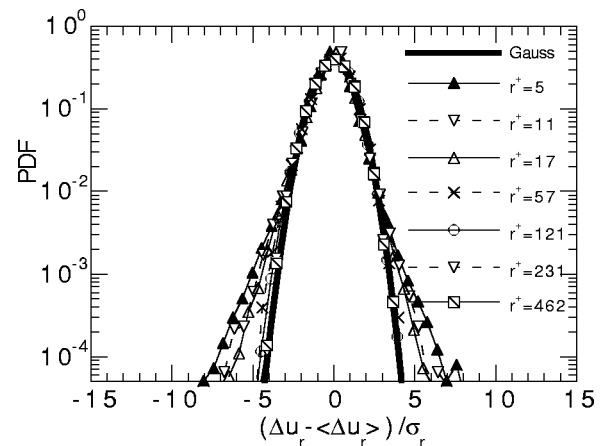

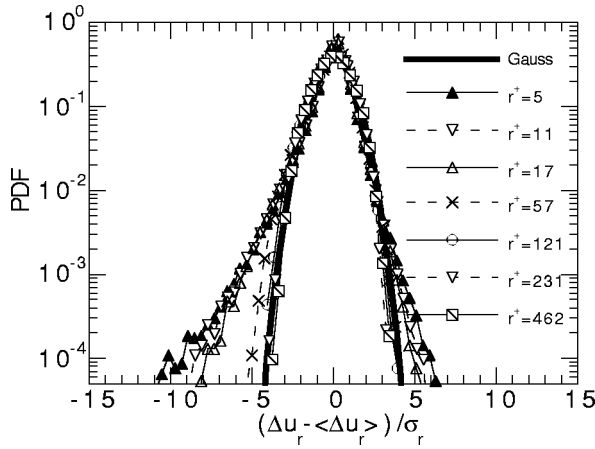FIG. 1. PDFs of the streamwise velocity differences (centerd at mean and renormalized by rms) at $y^+ = 218$.

FIG. 2. PDFs of the streamwise velocity differences (centered at mean and renormalized by rms) at $y^+ = 15$.

nential curve for smaller r^+. A slight asymmetry, consistent with the description of the homogeneous isotropic turbulence, is evident in all the PDFs shown in the figure. In Fig. 2 we report the PDF of the velocity difference at $y^+ = 15$. For small r^+ the PDFs are strongly asymmetric with long left tails, for large scales the behavior is quasi-Gaussian. In order to quantitatively investigate on the features of the asymmetry, we compute the skewness of the PDF for different values of the separation distance r^+. Results are shown in Fig. 3 where we plot the skewness factor for $y^+ = 15$ and $y^+ = 218$. The scale ranges from the minimum value allowed by the sampling frequency, up to $r^+ = 100$. In both cases the skewness is negative but close to the wall it reaches higher negative values in comparison to the data at the center of the channel. Furthermore, we have split the process in positive and negative differences and have computed their statistics separately. In Fig. 4 we show the skewness factor for $y^+ = 15$ and $y^+ = 218$ for positive and negative velocity differences. It is interesting to notice that, for positive differences, the skewness factor, up to measurement errors and for the considered scales, seems qualitatively independent from the y^+ coordinate.

In order to investigate in more details the difference between the positive and negative velocity differences we study the intermittency features by considering the ''plus'' and ''minus'' structure functions in the following way:
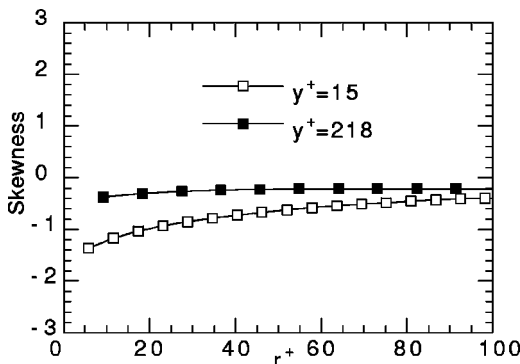

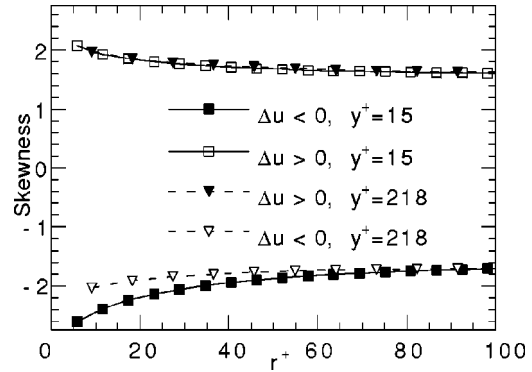FIG. 3. Skewness factor for different y^+ as a function of r^+.

FIG. 4. Skewness factor for different y^+ as a function of r^+ for negative and positive velocity differences.

$$S_p^{\pm}(r) = \left\langle \left[\frac{1}{2} (|\Delta u_r| \pm \Delta u_r) \right]^p \right\rangle, \qquad (1)$$

where $\Delta u_r = u(x+r) - u(x)$. A similar definition has been previously used in [9] and [10] for studying asymmetry of velocity increments in fully developed turbulence. As in Refs. [1–3] we have adopted the ESS and have analyzed the ESS local slopes. In Fig. 5 we consider the ESS local slopes for the fourth-order full structure function (dS_4/dS_3) at different measurement points. It is clear from the plot that close to the wall and at the center of the channel the relative scaling takes two different values: at the center of the channel the values are consistent with the homogeneous and isotropic turbulence $(\zeta_4 = 1.28)$; close to the wall the scaling exponent is much smaller and reaches the value of $\zeta_4 = 1.20$ or less (see [1]). In Fig. 6 we plot the local slope for the ''minus'' fourth-order structure function (dS_4^-/dS_3^-). The behavior of the ''minus'' structure function is very similar to the one computed with the full structure function, see Fig. 5. In Fig. 7 the local slope for the ''plus'' structure function is plotted as a function of r^+ at different distances from the wall. From the plot it appears that all lines are almost horizontal and are comprised between $dS_4^+/dS_3^+ = 1.26$ and 1.28. Two remarks are necessary at this point: (i) scaling exponents for the ''plus'' structure function are almost independent from the distance from the wall, (ii) the average value of the local slope for different y^+ is consistent with most of the experi-
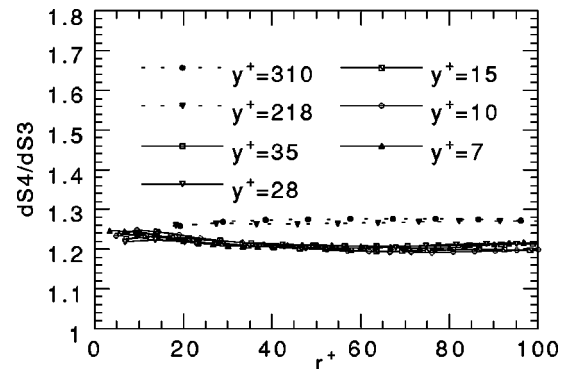

FIG. 5. ESS local slopes at different distances from the wall as a function of r^+.
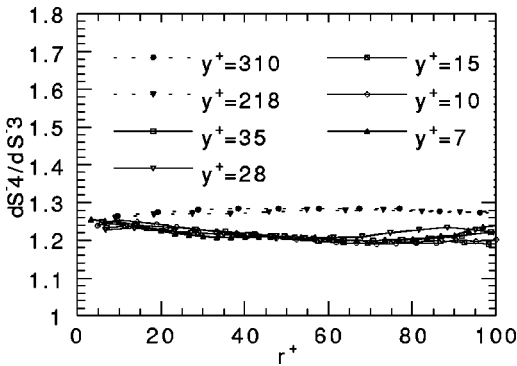
FIG. 6. ESS local slopes for the ''minus'' structure function at different distances from the wall as a function of r^+.

ments in homogeneous and isotropic turbulence. The same analysis on local slopes have been conducted on structure functions of different order: the results obtained are in agreement with the ones just presented and therefore are not reported.

In order to explain our findings, we briefly recall some of the features of the buffer layer dynamics. As previously mentioned, the near wall field is characterized by turbulent structures such as ''low'' and ''high'' speed streaks. These structures, discovered experimentally by flow visualisation by Kline et al. [11], consist of sinuous elongated regions whose velocity is respectively lower and higher than the mean one. Today it is well known that such structures are, on average, 1000 wall units long and have a spanwise wavelength of around 100 wall units. As explained in [12,13], their dynamic is strictly connected with the one of the streamwise vortices that populate the buffer region and is fundamental for the maintenance of turbulence in wall bounded flows. With simple conditional averaging techniques it is possible from numerical simulations or optical experimental methods to individuate the mean shape of a speed streak [14]. A sketch of a typical mean low speed structure in a x-z plane (x is the streamwise direction and z is the spanwise one) is shown in Fig. 8(a) (notice that the curved region is accentuated). Streamwise velocity gradients are located in front, in the back and wherever the streak has a knee (streaks are not perfectly straight). In Fig. 8(c), the strongest positive and negative velocity gradients, estimated in the regions indi-
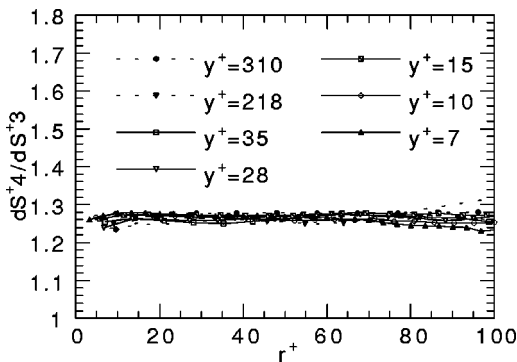

FIG. 7. ESS local slopes for the ''plus'' structure function at different distances from the wall as a function of r^+.
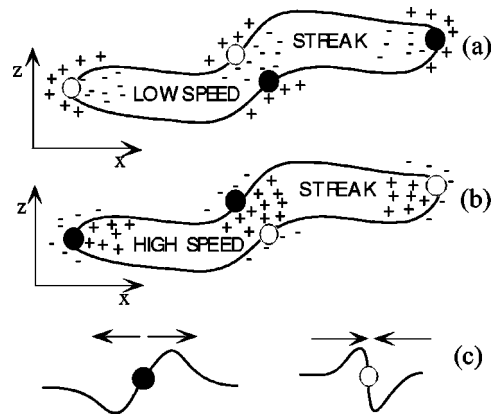

FIG. 8. Sketch of a low-speed streak (a) and a high-speed streak (b) with associated velocity gradients (c) estimated in points individuated by closed and open circles. $+$ and $-$ symbols indicate regions of positive and negative velocity fluctuations.

viduated by circles (open circles for negative gradients and closed circles for positive gradients), are also sketched. If at a hypothetical time $t=0$ the low speed streak would exhibit the same longitudinal velocity gradient (in absolute value) at its head and at its tail, because of the relative velocity of the streak itself with respect to the mean flow, at a successive time the gradient at the tail would increase its steepness forming structures which are *shocklike*, while at the head the gradient would decrease. This is also true in the region individuated by an open circle in Fig. 8(a) where the streak is not perfectly straight. In other words, a fluid particle located in the tail of a low speed streak moves slower than an upstream high-speed fluid particle, causing the distance between them to decrease in time, thereby creating a steeper gradient (see, for example, the ramp-model in [9]). For positive $\partial u/\partial x$ regions, the high-speed fluid downstream moves faster than the low-speed fluid upstream, so that $\partial u/\partial x$ decreases. Therefore, the highest longitudinal velocity gradients (absolute value) occur presumably more often at the tail of the low speed streak where $\partial u/\partial x<0$. An analogous argument can be applied to high speed streaks; see Fig. 8(b). As in the previous case, strong velocity gradients, which are now are located at the head of the structure, are more probable to be negative. As a consequence, both high- and low-speed streaks bring high negative velocity gradients, therefore higher negative velocity differences are more probable to occur with respect to the positive ones, giving rise to the skewed PDF for small scales (Fig. 2). For $y^+>70$ the streaky structure is almost absent (see, e.g., [14]), gradients are smoother and the statistics of the velocity differences are more similar to homogeneous and isotropic flows.

To summarize, we have found that the PDFs at small scales in the buffer region are strongly asymmetric. This asymmetry has been studied by the ''plus'' and ''minus'' structure functions, which highlighted that negative differences give a leading contribution to the scaling exponent of the full structure function: results from ESS statistics of the complete time series are comparable with the ones computed for only negative differences (see Figs. 5 and 6). Moreover the ESS statistics for the positive differences are in accor-

dance with the ones obtained in homogeneous and isotropic flows. We have related these statistical observations to the geometrical properties of the low- and high-speed streaks. ''Plus'' and ''minus'' structure functions provide a suitable tool for investigating nonhomogeneous and nonisotropic turbulent flows.

[1] M. Onorato, R. Camussi, and G. Iuso, Phys. Rev. E **61**, 29 (2000).

[2] F. Toschi, G. Amati, S. Succi, R. Benzi, and R. Piva, Phys. Rev. Lett. **82**, 25 (1999).

[3] R.A. Antonia, P. Orlandi, and G.P. Romano, Phys. Fluids **10**, 3239 (1998).

[4] R. Benzi, G. Amati, C.M. Casciola, F. Toschi, and R. Piva, Phys. Fluids **11**, 6 (1999).

[5] F. Toschi, E. Leveque, and G. Ruiz-Chavarria, Phys. Rev. Lett. **85**, 1436 (2000).

[6] R. Benzi, S. Ciliberto, R. Tripiccione, C. Baudet, F. Massaioli, and S. Succi, Phys. Rev. E **48**, 29 (1993).

[7] V. Carbone, L. Sorriso-Valvo, E. Martines, V. Antoni, and P. Veltri, Phys. Rev. E **62**, R49 (2000).

[8] I. Arad, L. Biferale, I. Mazzitelli, and I. Procaccia, Phys. Rev. Lett. **82**, 25 (1999); **82**, 5040 (1999).

[9] S.I. Vainshtein and K.R. Sreenivasan, Phys. Rev. Lett. **73**, 23 (1994); **73**, 3085 (1994).

[10] K.R. Sreenivasan, S.I. Vainshtein, R. Bhiladbala, I. San Gil, S. Chen, and N. Cao, Phys. Rev. Lett. **77**, 8 (1996); **77**, 1488 (1996).

[11] S.J. Kline, W.C. Reynolds, F.A. Schraub, and P.W. Runstadler, J. Fluid Mech. **30**, 741 (1967).

[12] J. Jeong, F. Hussain, W. Schoppa, and J. Kim, J. Fluid Mech. **332**, 185 (1997).

[13] J. Jimenez and A. Pinelli, J. Fluid Mech. **389**, 335 (1999).

[14] M. Gottero and M. Onorato, Eur. J. Mech. B/Fluids **19**, 23 (2000).